DISCRIMINATION CALIBRATION ANALYSIS METHODS FOR REGIONAL STATIONS

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ABSTRACT

For event discrimination, operational implementation of a regional seismic station requires three sequential calibration analyses. 1) Magnitude, distance, and amplitude corrections (MDAC) made to observed regional amplitudes are necessary so that what remains in the corrected amplitude is mostly information about the seismic source-type. Corrected amplitudes can be used in ratios to discriminate between earthquakes and explosions. Calibration of MDAC can be accomplished with empirical Bayes estimation, which naturally provides metrics to determine when adequate calibration data have been acquired, and provides statistical assurance that the errors associated with MDAC calibration are negligible in future operational discrimination analysis. 2) MDAC-corrected amplitudes can then be used in ratios to discriminate between earthquakes and explosions. However, there remain source effects such as those due to depth, focal mechanism, local material property and apparent stress variability that cannot easily be determined and applied as amplitude corrections. We have developed a mathematical model to capture these near source effects as random (unknown) giving an error partition of three sources: model inadequacy, station noise and amplitude correlation. This mathematical model is the basis for a general multi-station regional discriminant. Calibration analysis for the standard error of the discriminant includes the calculation of the variances of model inadequacy and station noise, and amplitude correlation. 3) Likelihood-based seismic event identification analysis with MDAC discriminants requires estimated source population means and covariance matrices for the discriminants from each of the possible source types used in our analysis (e.g., deep earthquake, shallow earthquake, and explosion). Anderson et al. (2007) note that source population covariance matrices and the pooled covariance matrix are best estimated element-wise to fully utilize available calibration events. We propose an algorithm that may be used to mildly adjust an element-wise covariance matrix to ensure positive definiteness and non-singular behavior. The algorithm uses the Frobenius norm as the calibration metric because it minimally adjusts the variance terms of an element-wise covariance relative to the off-diagonal covariance terms to achieve a stable, positive semi-definite covariance matrix. Importantly, we show that it is not necessary to propagate the errors of MDAC parameter estimates to final event identification analysis.

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OBJECTIVES

Our objective is to develop efficient methods to calibrate a regional seismic station for event discrimination. The formulation of the discriminant from a station requires amplitude corrections based on MDAC, the correct calculation of the discriminant standard error, and the estimation of a stable covariance matrix that describes the discriminants mathematical relationship with other discriminants.

Corrections to regional phases to account for magnitude, distance, and amplitude are necessary so that what remains is fundamentally information about seismic source type. MDAC requires calibration data for a station, which may not be available for a time after it becomes operational. In order to incorporate the station into event identification processing, MDAC calibration can be performed using an empirical Bayes approach. The approach allows the station to be quickly assimilated into the network and also provides natural metrics for determining when adequate calibration data have been acquired.

Once amplitudes have been corrected with MDAC, there are still near source effects which have not been accounted for in the model, such as effects due to depth, focal mechanism, local material properties, or apparent stress variability. The uncertainty introduced by these effects needs to be accounted for in the statistical formulation of a regional amplitude discriminant. The mathematical model presented below partitions total uncertainty into three sources: amplitude correlation, model inadequacy, and station noise. The model inadequacy partition represents the near-source effects and is handled statistically as a random component in the model.

The event classification matrix (ECM) [Anderson et al. (2007)] implements likelihood based discrimination, which requires an estimate of population means and covariance matrices for each source type population of interest. In order to take advantage of all possible calibration data, covariance matrices may be estimated element-wise, which does not guarantee numerical stability for ECM calculations. We present an algorithm that adjusts the covariance matrix in a way to ensure positive semi-definiteness while minimally affecting the variance terms in the matrix.

RESEARCH ACCOMPLISHED

MDAC Calibration

The ratio of regional P and S wave amplitude measurements at high frequencies can discriminate between earthquakes and explosions (e.g., Walter et al. (1995); Taylor (1996); Bottone et al. (2002)). An issue with using these amplitudes in a practical application is how to remove the effects due to path, site and magnitude to emphasize the source differences. In Taylor and Hartse (1998), Taylor et al. (2002) and Walter and Taylor (2002) the MDAC technique corrects each regional phase (e.g., Pn, Pg, Sn, Lg) amplitude as a function of frequency in an attempt to make amplitudes independent of distance, magnitude, and station. MDAC is a simple physically based model that accounts for propagation effects such as geometrical spreading and Q, and corrects observed amplitudes assuming the scaling of an earthquake spectral model developed by Brune (1970). The idea of using an earthquake MDAC model to correct amplitudes is that spectra from an explosion will exhibit a poor fit to the model, which will be apparent in an observed discriminant. Because of complex explosion source phenomenology it is not necessarily obvious which combinations of regional phases will best separate earthquake and explosion populations. The MDAC technique allows the formulation of any combination of regional phases in any frequency band, so that a diversity of discriminants can be explored. The MDAC model partitions regional seismic spectra into component parts. The instrument-corrected regional phase spectra can be thought of as a convolution between the source-type and the path. In the frequency domain, this can be mathematically represented as

$$A(\omega,\Delta) = S(\omega)G(\Delta)P(\omega)B(\omega,\Delta) \ , \tag{1}$$

where S is the source spectrum, G is geometrical spreading, P is the frequency-dependent site effect, and B is the anelastic attenuation with function arguments epicentral distance Δ and angular frequency ω. Here we have split the path effect into three components: 1) a frequency independent geometrical spreading component, 2) a range-independent and frequency-dependent site effect, and 3) an anelastic attenuation component. The logarithm of both sides of Equation 1 gives

$$\log A(\omega,\Delta) = \log S(\omega) + \log G(\Delta) + \log P(\omega) + \log B(\omega,\Delta). \tag{2}$$

To remove distance and magnitude trends in the data, we correct the observed spectrum $\log A_O(\omega,\Delta)$ so that

$$Y = \log A_C(\omega, \Delta) = \log A_O(\omega, \Delta) - \log A(\omega, \Delta), \tag{3}$$

where $A_C(\omega, \Delta)$ is the corrected spectrum. Equation 3 is used to calculate corrected MDAC amplitudes, denoted as Y, that are then used to construct discriminants. Specifically, from Equation 3, the corrected amplitude Y is a log observed amplitude minus MDAC—Y is the MDAC residual.

Referencing the development in Taylor and Hartse (1998), Taylor et al. (2002), and Walter and Taylor (2002), before MDAC is used in an operational setting, calibration earthquakes determine an average station site effect (Θ) [=unitless], an attenuation parameter (γ)[=unitless], and the average stress drop parameter (σ) [=Passcals], which are embedded in source spectrum (S) and anelastic attenuation (B) terms. It is important to recognize that the fundamental sources of error for MDAC corrections are the model inadequacy affecting all stations and individual station noise. Once the parameters are estimated from calibration earthquakes, the MDAC equation is treated as a known physical correction.

From equation 3, $\log A_C(\omega, \Delta)$ can reasonably be modeled as Gaussian with zero mean and variance τ^2. Here τ represents model uncertainty and station noise combined into one term, and only for earthquakes. The likelihood of n calibration earthquakes is

$$f\left[\text{data} | \Theta, \gamma, \sigma, \tau\right] = \prod_j \prod_{i=1}^{n} f\left[\log A_{C_i}(\omega_i, \Delta), M_{o_i} | \Theta, \gamma, \sigma, \tau\right], \tag{4}$$

where j indexes frequency, i indexes event, and M_{oi} is the event moment. In application reasonable estimates and bounds can be placed on Θ, γ, and σ from geophysical knowledge. The Bayesian formulation of these estimates are simply represented as prior probability density functions (PDFs) f(Θ), f(γ), and f(σ). Here, use a Uniform(l, u) PDF in each case with l and u specified from geophysical knowledge. The prior PDF f(τ) is also modeled as Uniform. More sophisticated, physically based prior for MDAC parameters could be developed with further research. The full likelihood is now

$$f\left[\Theta, \gamma, \sigma, \tau, \text{data}\right] = f(\Theta)f(\gamma)f(\sigma)f(\tau)f\left(\text{data} | \Theta, \gamma, \sigma, \tau\right) \tag{5}$$

and the posterior PDF is

$$f\left[\Theta, \gamma, \sigma, \tau, | \text{data}\right] = c(\text{data})f(\Theta)f(\gamma)f(\sigma)f(\tau)f\left(\text{data} | \Theta, \gamma, \sigma, \tau\right), \tag{6}$$

where c(data) is a constant that ensures integration to unity. The uniform priors simply ensure that an MDAC parameter's range of possible values agree with physical basis. Several possible values for Θ, γ, σ and τ can be calculated, each derived from Equation (6), the posterior PDF. We recommend the mode of the posterior PDF as MDAC calibration values which are conceptually values of Θ, γ, σ, and τ that are the most probable given the earthquake calibration data.

The data analyzed as an example are events at the Nevada Test Site (NTS) observed with combinations of four seismic stations: Kana, Utah, (KNB); Elko, Nevada (ELK), Landers, California (LAC) and Columbia College, California (CMB). Observed amplitudes are RMS measurements converted to pseudo spectra by application of Parseval's theorem [see Appendix B of Taylor et al. (2002)] with a 6 to 8 Hertz filter window (so that the number of frequencies is j = 1). KNB observed 43 earthquakes; these events are used to illustrate the Bayesian calculations of MDAC parameters for the Lg phase. Posterior PDFs are shown in Figure 1. Note the small amount of variability (spread) in the PDFs for Θ and γ. The slightly higher variability in the PDF for σ is because the spectrum in this example was constrained by high-frequency amplitudes and the event moment. With lower frequency amplitudes included in the analysis, we expect the variability of the posterior PDF for σ to be similar to that for Θ and γ. This will be demonstrated in a follow-on comprehensive analysis.
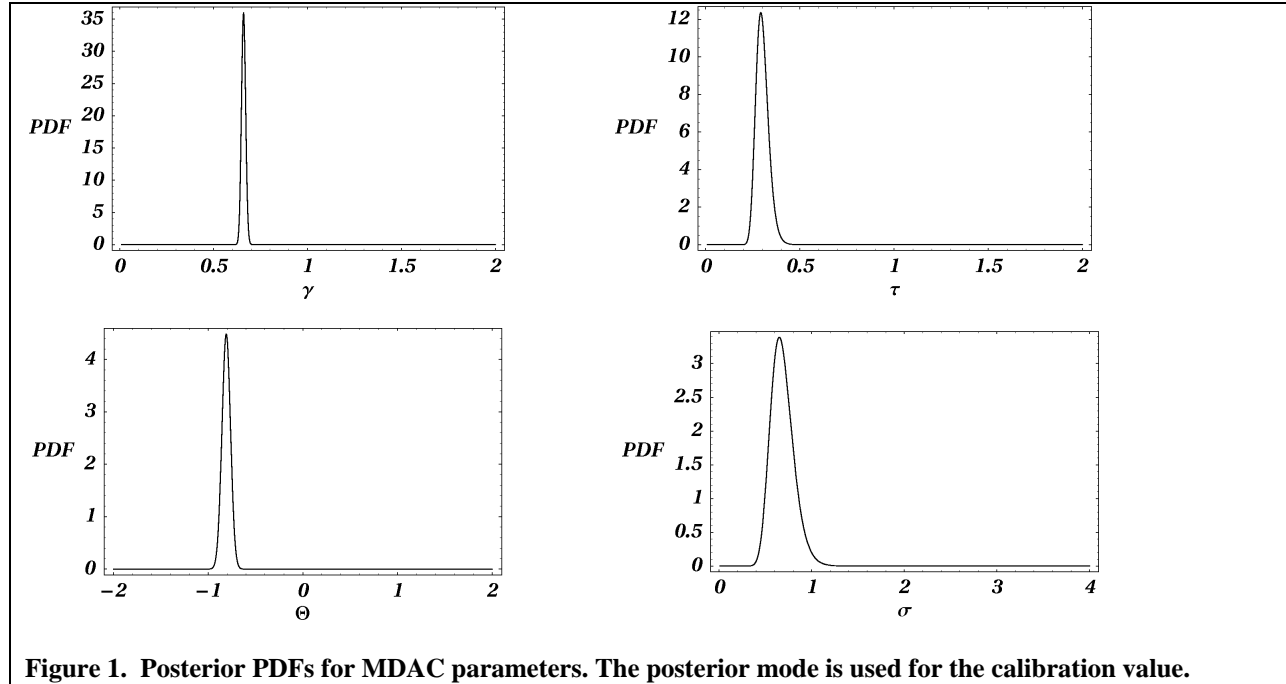
Figure 1. Posterior PDFs for MDAC parameters. The posterior mode is used for the calibration value.

Discriminant Errors: MDAC Model Inadequacy and Station Noise

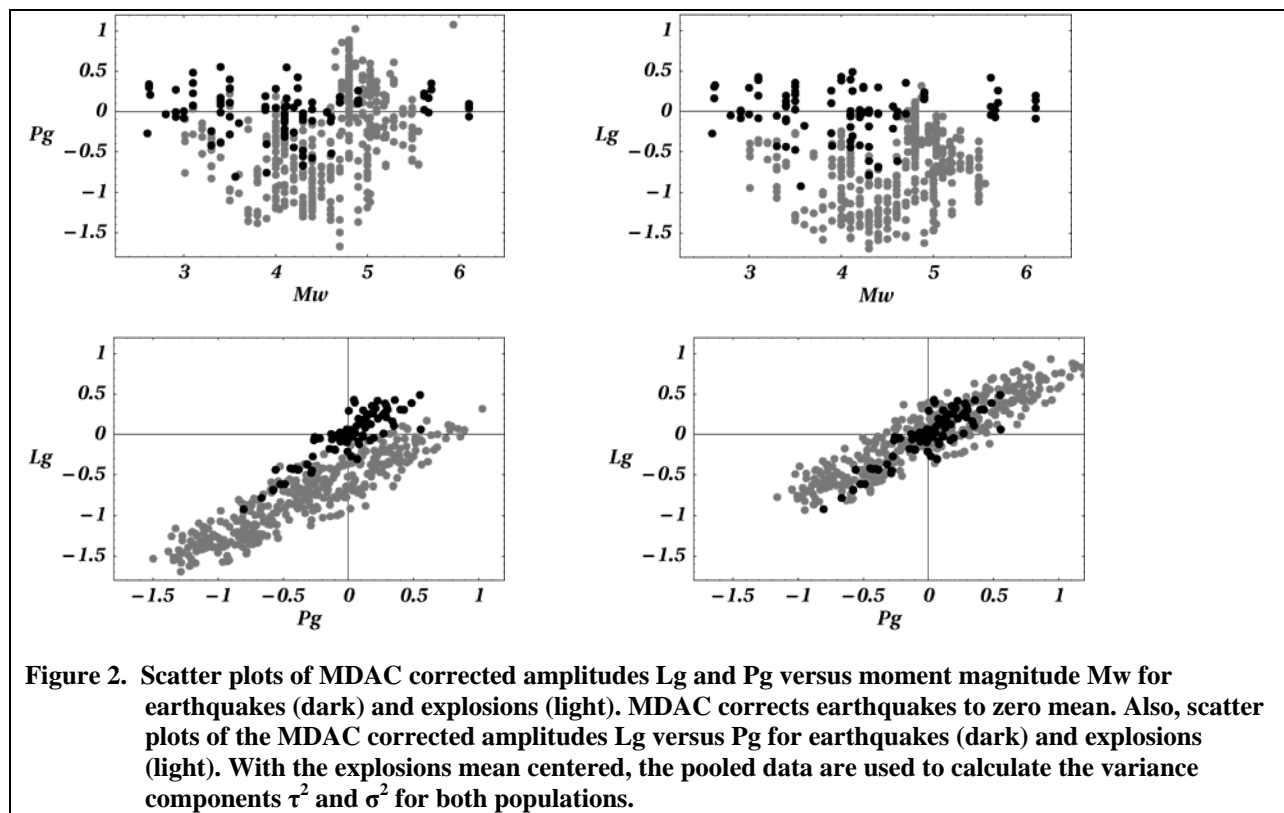The conceptual representation of the MDAC amplitude model is

$$Y = \log(\text{corrected amplitude}) = Bias(\text{source - type}) + Event + Noise , \qquad (7)$$

where *Bias*(source-type) is a source-type constant, *Event* is a zero mean random effect with variance τ^2 that varies from event to event and represents model inadequacy from effects such as depth, focal mechanism, local material property and apparent stress variability, and *Noise* represents measurement and ambient noise, also with zero mean and variance σ^2. Note that in this section, calibration analysis partitions the variance τ^2 in the MDAC calibration analysis above into two variance components *Event* (τ^2) and *Noise* (σ^2). The MDAC approach results in a *Bias* term for earthquakes that is near zero, whereas for explosions the *Bias* is non-zero indicating discrimination potential. The error terms *Event* and *Noise* are modeled as equal for both explosions and earthquakes; therefore pooled data (amplitudes from explosions and earthquakes with their respective population means subtracted) are used in the calculation of τ^2 and σ^2.

The station-averaged corrected amplitude $\overline{Y} = \underline{1}^T Y / n$ has a standard error $\tau^2 + \sigma^2/n$, where n is the number of station amplitudes used in the average. Note that forming regional discriminants from station-averaged corrected amplitudes exactly parallels the methodology of the m_b versus M_s discriminant where both are station-averaged magnitudes. Omitting the term *Event* in Equation (7) implies that the corrected amplitude at a station is *Bias* plus station noise. As demonstrated with the following argument, this model formulation is fundamentally inconsistent with the realities of seismic observation. The standard error of $\overline{Y}$ with E removed from Equation (7) is σ^2/n ($\tau^2 = 0$) and decreases as the number of stations n observing an event increases. This implies that if enough stations observe an event, this standard error effectively goes to zero and the average corrected amplitude quickly converges to Bias implying near-perfect discrimination capability. By not including the term *Event*, effects such as depth, focal mechanism, local material property and apparent stress variability are not accounted for in the theoretical model of amplitude, and clearly these effects cannot be removed by station averaging. The model given by Equation (7) captures these local source effects by admitting that they cannot be mathematically (theoretically) represented. Treating local source effects as a random effect *Event* compensates for them as a component in the standard error of a discriminant. Also, the lower bound of standard error $\tau^2 + \sigma^2/n$ is non-zero and therefore consistent with realistic seismic monitoring. Another important property of this model is that a corrected amplitude for a single event is correlated across stations. The correlation $(\tau^2/(\tau^2+\sigma^2))$ implies that large *Event* adjustment increases correlation

between stations because this random adjustment is applied to all stations observing an event, that is, the stations stochastically move together. Small *Event* adjustment implies the correction model is good and is conceptually equivalent to stations with incoherent noise. Small Event adjustment also implies τ^2 is small and the standard error of $\bar{Y}$ is reduced further through station averaging.

The data used to illustrate the calculation of the variance components τ^2 and σ^2 are events at and surrounding the Nevada Test Site (NTS). Events were observed with combinations of four seismic stations: Kanab, Utah (KNB); Elko, Nevada (ELK), Landers, California (LAC) and Columbia College, California (CMB). MDAC amplitudes from these stations are Pg and Lg amplitudes from pseudo spectral measurements with a 6 to 8 Hertz filter window. After applying data quality metrics (e.g., signal to noise and removal of events within 100 kilometers of a station), the data table consisted of 41 earthquakes (EQ) and 159 explosions (EX) for a total of 200 events. Moment magnitudes ranged from 2.6 to 6.1. Amplitude corrections for discrimination remove the effects of magnitude, source scaling and distance so that what remains in the corrected amplitude is fundamentally information about source type. Figure 2 demonstrates the removal of the effect of moment magnitude from the Pg and Lg amplitudes with MDAC. Also, Figure 2 shows the explosions and earthquake amplitudes before and after removal of population means – these centered data are used to calculation the variance components τ^2 and σ^2, and additionally the correlation between amplitudes in a discriminant. The fit residuals are given in Figure 3 and the calculated variance components are given Table 1.

Figure 2. Scatter plots of MDAC corrected amplitudes Lg and Pg versus moment magnitude Mw for earthquakes (dark) and explosions (light). MDAC corrects earthquakes to zero mean. Also, scatter plots of the MDAC corrected amplitudes Lg versus Pg for earthquakes (dark) and explosions (light). With the explosions mean centered, the pooled data are used to calculate the variance components τ^2 and σ^2 for both populations.

Figure 3. Fitted model residuals for Event and Noise using pooled explosions and earthquake data. Residuals indicate reasonable agreement with model assumptions of uncorrelated terms Event and Noise.
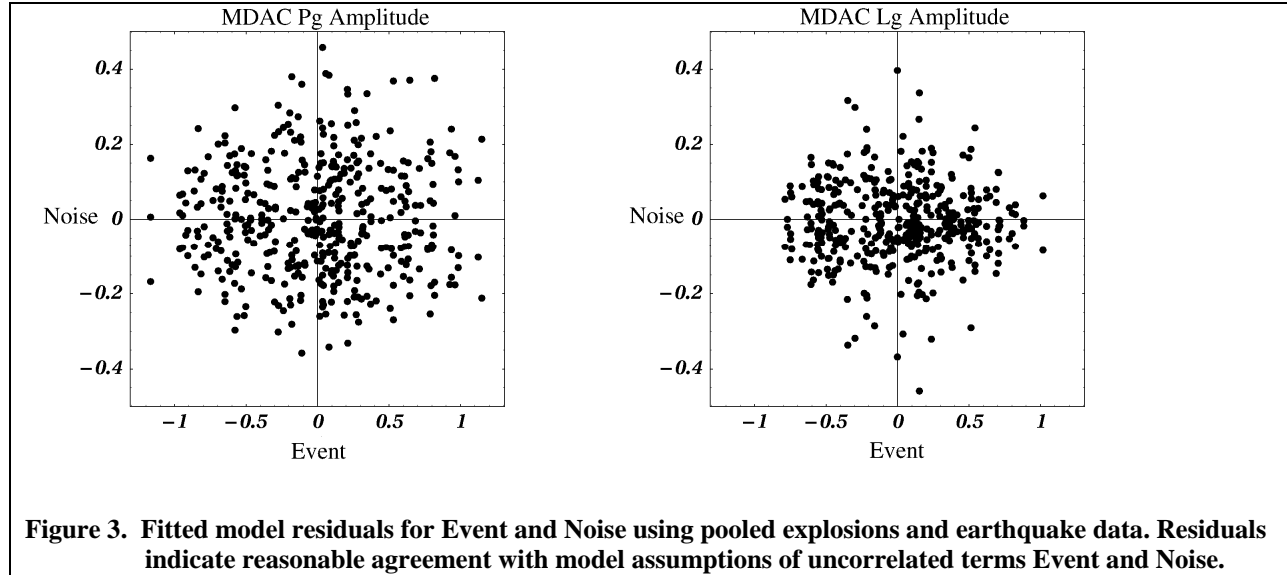
Table 1. Calculated variance components τ^2 and σ^2. From the pooled data presented in Figure 2, the calculated correlation between Pg and Lg is 0.95.

	Event (τ^2)	Noise (σ^2)
Pg	0.23	0.04
Lg	0.16	0.02

Structured Covariance Matrix of Discriminants

Seismic event identification using regularized discrimination requires estimated source population means and covariance matrices for the discriminants from each of the possible source types (e.g., deep earthquake, shallow earthquake, or explosion). Anderson et al. (2007) notes that the estimated source population covariance matrices denoted S_k, k = 1, 2... K, and the pooled covariance matrix, S_0, are estimated element-wise in order to take advantage of all available calibration data. Due to few calibration events or strongly correlated discriminants, one or more S_k and/or S_0 may be singular. We present an algorithm inspired by Shaw and Geyer (1997) that may be used to adjust elements of singular covariance matrix estimates prior to regularized discriminant analysis (RDA).

Anderson et al. (2007) uses standardized discriminants Y to solve the seismic identification problem. We assume that the k populations are independent and, for the kth source population, $Y \sim MVN_p (\mu_k, \Sigma_k)$; where μ_k and Σ_k are the source mean vector and covariance matrix, respectively, and p discriminants are used. The log likelihood function for μ_k and Σ_k, given the data y is

$$l(\mu_k, \Sigma_k | y) = -\frac{n}{2} \log|2\pi\Sigma_k| - \frac{n}{2} \operatorname{tr}\left(\Sigma_k^{-1} S_k\right) - \frac{n}{2} (\bar{y} - \mu_k)^T \Sigma_k^{-1} (\bar{y} - \mu_k),$$

(1)

where tr($\cdot$) denotes the trace operator and $S = \frac{1}{n}\left[\sum_{i=1}^{n} (y_i - \bar{y})(y_i - \bar{y})^T\right]$. Maximum likelihood estimators (MLEs) for

μ_k and Σ_k are $\bar{y}_k$ and S_k, respectively. These estimators are derived by setting the partial derivatives of equation (1) equal to zero and then solving for μ and Σ. The framework in Anderson et al. (2007) can be carried out as described if S_k is non-singular. If S_k is singular, however, a different estimator of Σ_k is required. We desire an estimator, say S_k^*, that is positive semi-defnite, as "close" to S_k as possible, and that the variance terms be minimally different from those in S_k compared to the covariance terms.

We choose the Frobenius norm as the one to minimize, because as we demonstrate here, it is a norm which allows us to minimally alter the variance terms of S compared with the covariance terms. The Frobenius norm, denoted $\|\cdot\|$, is defined as the square-root of the sum of the absolute value of the matrix elements. Our optimization criterion becomes

$$\min_{Q}\|S - Q\| = \sqrt{\sum_{i=1}^{p}\sum_{j=1}^{p}\left|s_{ij} - q_{ij}\right|^2} \tag{2}$$

where Q is a $p \times p$ symmetric, invertible matrix.

Because S is a symmetric matrix, $S = S^T$, the eigenvalues of S are real and associated eigenvectors are orthogonal. Let the eigen decomposition of S be $V\Lambda V^T$, where Λ is the diagonal matrix of ordered eigenvalues ($\lambda_i \geq \lambda_j$ for all i < j) and the columns of V are the corresponding eigenvectors. Likewise, let $Q = UCU^T$, where $C = \text{diag}\{c_1 \geq c_2 \geq ... \geq c_p\}$ and U is the matrix whose columns are the corresponding eigenvectors. We then want to minimize

$$\left\|V\Lambda V^T - UCU^T\right\|. \tag{3}$$

A linear algebra identity can be used to show that $\left\|V\Lambda V^T - UCU^T\right\| \leq \left\|V^2\right\| \left\|\Lambda - C\right\|$. Both Λ and C are diagonal matrices, where the diagonal terms are sorted from highest to lowest and all of the elements are real. Therefore the diagonal vector $\boldsymbol{\lambda}$ has a set of positive, zero, and negative values, as does the vector $\mathbf{c}$. The Frobenius norm can be written as the sum of terms partitioned into positive, zero and negative terms so that

$$\|\Lambda - C\| = \sqrt{\sum_{i=1}^{m}\left|\lambda_i - c_i\right|^2 + \sum_{i=m+1}^{n}\left|\lambda_i - c_i\right|^2 + \sum_{i=n+1}^{p}\left|\lambda_i - c_i\right|^2}, \tag{4}$$

where $\{\lambda_1, \lambda_2, ..., \lambda_m\} > 0$, $\{\lambda_{m+1}, ..., \lambda_n\} = 0$ and $\{\lambda_{n+1}, ..., \lambda_p\} < 0$. By choosing

$$c_i = \begin{cases} \lambda_i & i = 1, 2, ..., n \\ 0 & i = n+1, n+2, ... p \end{cases}$$

Equation 4 becomes $\|\Lambda - C\| = \sqrt{\sum_{i=n+1}^{p}\left|\lambda_i - c_i\right|^2}$ and consequently, $\left\|V\Lambda V^T - UCU^T\right\| \leq \left\|V^2\right\|\sqrt{\sum_{i=n+1}^{p}\left|\lambda_i - c_i\right|^2}$.

Our estimate of S_k is then $Q = VCV^T = V\text{Diag}\{\lambda_1, \lambda_2, ..., \lambda_m, 0, 0, ..., 0\}V^T$.

The following example shows how the algorithm works. Figure 4 shows the original eigenvalues of the structured covariance matrix for a data set of deep earthquakes S_{DEQ}, which is singular. Figure 5 shows how S_{DEQ} is adjusted to result in S^*_{DEQ}, a non-singular matrix. The white areas of Figure 5 indicate no change in the imposed structure. Notice that the diagonal elements are changed minimally, compared to the off-diagonal elements. This preserves the variance structure of the individual discriminants while minimally adjusting the covariance terms to achieve invertibility.
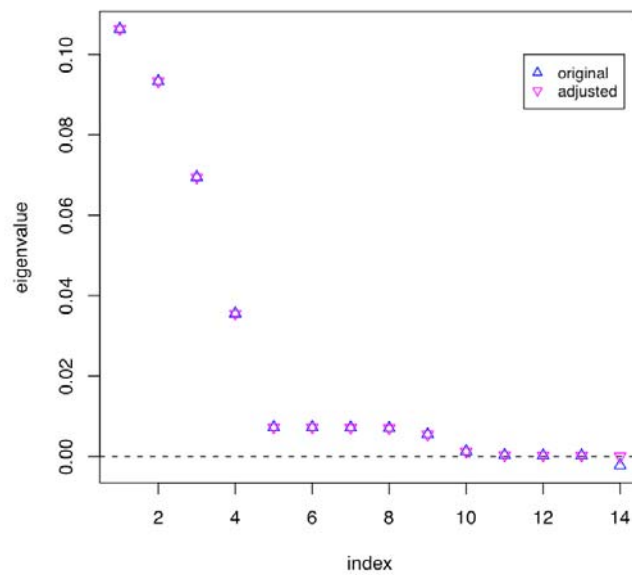
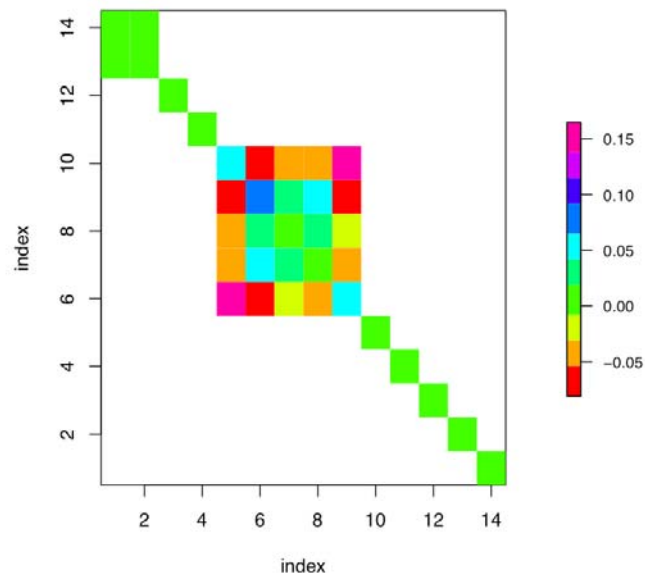**Figure 4. Eigenvalues of the original covariance matrix S_k and adjusted covariance S^*_k.**

Figure 5. Percent adjustment to S_k to achieve invertibility.

CONCLUSIONS AND RECOMMENDATIONS

Three core conclusions can be stated from our analysis. First, MDAC parameters can be derived with sufficient precision to view the MDAC correction as fixed and deterministic in event identification analysis which implies it is not necessary to propagate the error from MDAC parameter estimation in event identification analysis. Further, reasonable physical constraints can be placed on MDAC parameters with Bayesian theory to aid in the calculation of MDAC parameters. We believe that no more than 100 well chosen earthquakes are necessary to calibrate the MDAC correction equation, and research and demonstration analysis for FY09 will verify this conjecture. Second, there are

two important sources of error when constructing a discriminant with MDAC correction amplitudes – the error due to MDAC model inadequacy and station noise. The calibration analysis for deriving these variance components is accomplished by embedding MDAC theory into a statistical random-effects linear model. The resulting standard error of a discriminant from this model correctly reduces station noise though averaging, but requires improvements to physical correction theory (an improved MDAC model) to reduce model inadequacy error. Third, the covariance matrices necessary to combine a diversity of discriminants in event identification analysis can be calculated with all available data by individually calculating each element of a matrix (variance and covariance elements). However, these calculations, while giving a symmetric covariance matrix, do not guarantee that the matrix is positive definite. We have demonstrated that with the Frobenius norm, a covariance matrix constructed element-by-element can be adjusted, with little impact on the variance terms, to achieve positive definiteness.

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